Version 2 on pages 3-4

Math 230-01 (Ellis) Spring 2009 Quiz 1 **Name:** KEY

Instructions. Clearly circle the correct answer. Time limit is *exactly* 20 minutes.

1. Suppose you are examining a statement of the form $\exists x(\neg P(x) \wedge Q(x))$. If you are trying to prove that the statement is true, you need to find a value of x such that

   (a) $P(x)$ is true and $Q(x)$ is true
   (b) $P(x)$ is true and $Q(x)$ is false
   (c) $P(x)$ is false and $Q(x)$ is true $\rightarrow$ $\neg$False $\wedge$ True = True $\wedge$ True
   (d) $P(x)$ is false and $Q(x)$ is false = True

2. Circle the letter of the correct proposition.

   (a) The implication $r \rightarrow s$ is logically equivalent to the inverse of $r \rightarrow s$.
   (b) The inverse of the implication $r \rightarrow s$ is logically equivalent to the converse of $r \rightarrow s$.
   (c) The inverse of the implication $r \rightarrow s$ is logically equivalent to the contrapositive of $r \rightarrow s$.
   (d) The contrapositive of the implication $r \rightarrow s$ is logically equivalent to the converse of $r \rightarrow s$.

   inverse: $\neg r \rightarrow \neg s$
   converse: $s \rightarrow r$.

r	s	$\neg r$	$\neg s$	$\neg r \rightarrow \neg s$	$s \rightarrow r$
T	T	F	F	T	T
T	F	F	T	T	T
F	T	T	F	F	F
F	F	T	T	T	T

3. Express in symbols the compound proposition
   "I don't swim in the ocean when the wind is blowing,"
   assuming that b and f are the following propositions:
   b: "I swim in the ocean."
   f: "The wind is blowing."

   (a) $\neg f \rightarrow b$
   (b) $\neg b \rightarrow f$
   (c) $b \rightarrow f$
   (d) $f \rightarrow \neg b$

   see p. 6 for "q when p"
   $p \rightarrow q$

   $\forall r$ $\forall s$ if $(r>0 \wedge s>r)$

4. Which of these statements says that if a number is positive and a second number is greater than the first number, then the second number is also positive?

   (a) $\forall r \exists s((r > 0) \rightarrow [(s > 0) \wedge (s > r)])$ then $s>0$
   (b) $\forall r \forall s((r > 0) \wedge (s > r))$
   (c) $\forall r \exists s((r > 0) \rightarrow (s > 0))$
   (d) $\forall r \forall s([(r > 0) \wedge (s > r)] \rightarrow s > 0)$

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5. Suppose you wish to prove the following about integers x and y by contraposition (contrapositive).

If $x - y$ is odd, then x is even or y is even.

Circle the letter of the hypothesis that you will begin with.

(a) x is even or y is even
(b) x is odd, y is odd, and $x - y$ is odd
(c) x is odd and y is odd $\leftarrow$ negation of x is even $\vee$ y is even
(d) $x - y$ is odd
(e) $x - y$ is even

6. Circle the letter corresponding to the negation of

$$\exists x \forall y \exists z (P(x, y) \oplus Q(x, z))$$

(a) $\forall x \exists y \forall z(\neg P(x, y) \oplus \neg Q(x, z))$
(b) $\exists x \forall y \exists z(P(x, y) \leftrightarrow Q(x, z))$
(c) $\forall x \exists y \forall z(P(x, y) \leftrightarrow Q(x, z))$
(d) $\exists x \forall y \exists z(\neg P(x, y) \oplus \neg Q(x, z))$
(e) $\forall x \exists y \forall z(P(x, y) \oplus Q(x, z))$

$\neg(p \oplus q) \equiv p \leftrightarrow q$

p	q	$p \leftrightarrow q$	$p \oplus q$
T	T	T	F
T	F	F	T
F	T	F	T
F	F	T	F

7. Suppose you wish to give a direct proof of
"If $x + 3$ is an odd integer, then $x + 1$ is an odd integer."
Which of these statements do you assume?

(a) $x + 1 = 2k + 1$
(b) $x + 1 = 2k$
(c) $x + 3 = 2k + 1$
(d) $x + 3 = 2k$

Assume $x+3$ is odd.
Then $x+3 = 2k+1$ for some $k \in \mathbb{Z}$.

8. Select the negation of the following statement:
"All students in the club except Ed are officers."

possible translation: $\neg O(Ed) \wedge \forall x (x \neq Ed \rightarrow O(x))$
where $O(x) =$ "x is an officer".

(a) Some student in the club other than Ed is not an officer, and Ed is not an officer.
(b) Some student in the club other than Ed is not an officer, or Ed is an officer.
(c) Ed and some other student in the club are officers.
(d) Ed is the only student in the club that is an officer.

Negate to get $O(Ed) \vee \exists x (x \neq Ed \wedge \neg O(x))$,
which translates to (b).

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Instructions. Clearly circle the correct answer. Time limit is *exactly* 20 minutes.

1. Suppose h and c are these propositions:

   h: "I ride my bicycle."

   c: "The sky is cloudy."

   Express in symbols the compound proposition

   "I don't ride my bicycle when the sky is cloudy."

   (a) $h \to c$
   (b) $c \to \neg h$
   (c) $\neg c \to h$
   (d) $\neg h \to c$

   See "q when p" page 6
   $p \to q$

2. Circle the letter corresponding to the negation of

   $$\exists x \forall y \exists z (P(x,y) \leftrightarrow Q(x,z))$$

   (a) $\forall x \exists y \forall z (P(x,y) \oplus \neg Q(x,z))$
   (b) $\exists x \forall y \exists z (P(x,y) \leftrightarrow Q(x,z))$
   (c) $\forall x \exists y \forall z (P(x,y) \leftrightarrow Q(x,z))$
   (d) $\exists x \forall y \exists z (\neg P(x,y) \oplus \neg Q(x,z))$
   (e) $\forall x \exists y \forall z (P(x,y) \oplus Q(x,z))$

   $\neg(p \leftrightarrow q) \equiv p \oplus q$.
   check by truth table

3. You wish to prove the following about integers m and n by contraposition (contrapositive):

   If m is odd and n is odd, then $m - n$ is even.

   Which hypothesis would you begin with?

   (a) m is odd and n is odd
   (b) $m - n$ is odd ← negation of m-n even.
   (c) m is even or n is even
   (d) $m - n$ is even
   (e) m is odd, n is odd, and $m - n$ is odd

4. Suppose you are examining a statement of the form $\forall x(P(x) \vee \neg Q(x))$. If you are looking for a counter example, you need to find a value of x such that

   (a) $P(x)$ is true and $Q(x)$ is true
   (b) $P(x)$ is true and $Q(x)$ is false
   (c) $P(x)$ is false and $Q(x)$ is true ←
   (d) $P(x)$ is false and $Q(x)$ is false

   $\neg(P(x) \vee \neg Q(x)) \equiv \neg P(x) \wedge Q(x)$
   ↑
   which is true when

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5. Which of the following propositions is correct?

(a) The inverse of the implication $p \rightarrow q$ is logically equivalent to $p \rightarrow q$.
(b) The converse of the implication $p \rightarrow q$ is logically equivalent to the inverse of $p \rightarrow q$.
(c) The contrapositive of the implication $p \rightarrow q$ is logically equivalent to the inverse of $p \rightarrow q$.
(d) The converse of the implication $p \rightarrow q$ is logically equivalent to the contrapositive of $p \rightarrow q$.

converse of $p \to q$ is $q \to p$
inverse of $p \to q$ is $\neg p \to \neg q$ take contrapositive to show $q \to p$ is logically equivalent.

$\forall x$ $\forall y$

6. Which of these statements says that if a number is positive and a second number is greater than the first number, then the second number is also positive?

(a) $\forall x \exists y((x > 0) \rightarrow (y > 0))$
(b) $\forall x \forall y([(x > 0) \wedge (y > x)] \rightarrow y > 0)$
(c) $\forall x \forall y((x > 0) \wedge (y > x))$
(d) $\forall x \exists y((x > 0) \rightarrow [(y > 0) \wedge (y > x)])$

if $(x > 0 \wedge y > x)$
then $y > 0$.

7. Suppose you wish to give a direct proof of

"If $n + 1$ is an odd integer, then $n + 3$ is an odd integer."

Which of these statements do you assume?

(a) $n + 3 = 2k$
(b) $n + 3 = 2k + 1$
(c) $n + 1 = 2k$
(d) $n + 1 = 2k + 1$

Direct proof:
Assume $n+1$ is odd.
Then $n+1 = 2k+1$ for some $k \in \mathbb{Z}$.

Define $A(x)$ = "x charges admission"

translate:
$\neg A(\text{Lincoln Park}) \wedge \forall x (x \neq \text{Lincoln Park} \to A(x))$

8. Which of the following is the negation of the statement:

"Every zoo in the United States except the Lincoln Park Zoo charges admission."

(a) The Lincoln Park Zoo and some other zoo in the United States charge admission.
(b) The Lincoln Park Zoo is the only zoo in the United States that charges admission.
(c) Some zoo in the United States other than the Lincoln Park Zoo does not charge admission, or the Lincoln Park Zoo charges admission.
(d) Some zoo in the United States other than the Lincoln Park Zoo does not charge admission, and the Lincoln Park Zoo does not charge admission.

negation: $A(\text{Lincoln Park}) \vee \exists x (x \neq \text{Lincoln Park} \wedge \neg A(x))$,
which translates to c.