PRINT Last name:____________________ First name:____________________

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Math 230 Exam 2, Spring 2009

(Use this page and back page for extra workspace.)

I. Short answer 1-12. Do not give proofs (3pts each).

1. Compute the following:

   28 div 3 = __________ 28 mod 3 = __________

   -17 div 6 = __________ -17 mod 6 = __________

2. What condition on positive integers a and b must hold for them to be *relatively prime*?

   ____

3. Give an example of a function $f : \mathbb{R} \to \mathbb{R}$ that is one-to-one but not onto.

4. Define $f : \mathbb{R} \to \mathbb{R}$ by $f(x) = x^3$, and $g : \mathbb{R} \to \mathbb{R}^+$ by $g(x) = 2^x$. Evaluate the following:

   $(g \circ f)(-1) =$ ______________

   $(g \circ f)^{-1}(16) =$ ________________

5. The first and second terms of an arithmetic progression are $a_1 = 5.5$ and $a_2 = 8$. What is a_4?

   $a_4 =$ __________

6. Assume for two positive integers a and b that $a \cdot b = 2^4 \cdot 3^3 \cdot 5 \cdot 7^2$ and $\gcd(a, b) = 2^2 \cdot 3 \cdot 7$. What is $\operatorname{lcm}(a, b)$?

   ____

7. Compute the following:

   $\gcd(0, 4) =$ __________ $\gcd(-12, -50) =$ __________

8. Order the following expressions in terms of **increasing** growth rate: n^2, $n!$, $n\log_2 n$, 2^n, $\log_2 n$

   ________ ________ ________ ________ ________

9. In the best big-oh notation, how many multiplications are needed for Horner's method to evaluate a polynomial of degree n at a constant c?

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   <u>Horner's method</u>
   procedure $Horner(c, a_0, a_1, \ldots, a_n$: real numbers)
   $y := a_n$
   for $i := 1$ **to** n
   $\quad y := y * c + a_{n-i}$

10. Name an algorithm we studied for which the best-case time complexity is different than the worst-case time complexity (in terms of the representative operation we discussed).

    ____

11. Circle the prime numbers and only the prime numbers in this list:

    73, 92, 93, 109, 127, 133

12. A 2 cent and a 5 cent stamp are available (without limit) to make postage. What is the smallest amount of postage N in cents such that every $N, N+1, N+2, N+3, \ldots$ can be composed of 2 cent and 5 cent stamps?

    $N =$ ________

II. Computation Problems 13-16 (10pts each). For full credit, show work to clearly justifying your answer.

13. Use the Euclidean Algorithm to express $\gcd(222, 180)$ as $r \cdot 222 + s \cdot 180$ for some $r, s \in \mathbb{Z}$.

14. Compute the sum $\sum_{i=0}^{10} \left(2 \cdot 3^{i+2} - 4 \cdot i\right)$ (an unsimplified answer is acceptable but must not contain ellipses $(\cdots)$ or i).

15. Compute $(60032 \cdot 24005 + 90511 \cdot 3030) \mod 3$. Shortcuts are recommended, but must be clear from your work.

16. Let $f(x) = 5x^2 + 10\log_2 x$. Compute witnesses C and k that show $f(x)$ is $O(x^2)$. (Hint: $x > k \rightarrow |f(x)| \leq C|x^2|$.)

III. Proofs 17-18 (12pts each). Partial credit for good proof structure.

17. Prove the following. Let m, n be positive integers greater than 1, and let a, b be integers. If $n | m$ and $a \equiv b \pmod{m}$, then $a \equiv b \pmod{n}$.

18. Prove by a careful induction argument that every positive integer can be written in the form $2^r \cdot m$, where r is an integer and m is an odd integer.

[WORKSPACE]