PRINT Last name:____________________ First name:____________________

Signature:____________________ Student ID:____________________

Math 230 Exam 1, Spring 2009

1. (8pts) For which rows of the truth table is the compound proposition $(s \leftrightarrow t) \rightarrow (s \oplus r)$ false?

r	s	t	
T	T	T	
T	T	F	
T	F	T	
T	F	F	
F	T	T	
F	T	F	
F	F	T	
F	F	F	

2. (6pts) The original statement is "If $1 + 1 = 2$ then $2 + 2 = 3$." Circle the correct truth value of each of the following statements:

   Contrapositive of the original statement (True / False)

   Converse of the original statement (True / False)

   Inverse of the original statement (True / False)

3. (8pts) Determine whether or not $(r \rightarrow s) \rightarrow t$ is logically equivalent to $r \rightarrow (s \rightarrow t)$. Show your work or carefully describe your argument.

4. (8pts) Among these 5 propositions are exactly 1 tautology and exactly 1 contradiction. Write $\mathbb{T}$ next to the tautology. Write $\mathbb{F}$ next to the contradiction. Do nothing for the rest of the propositions.

   ____**(a)** $r \wedge s \wedge t$

   ____**(b)** $(s \wedge t) \vee (r)$

   ____**(c)** $(p \leftrightarrow q) \vee (p \leftrightarrow \neg q)$

   ____**(d)** $p \wedge (p \vee \neg q) \wedge \neg p$

   ____**(e)** $\neg r \rightarrow r$

5. (4pts) Write the negation of the following statement (Do not write "It is not the case that ...").
   "I will ride my bike or drive my car but not both."

6. (4pts) Is the following argument valid? (Circle Yes / No)

$$\begin{array}{r} \neg r \rightarrow s \\ \neg s \\ \hline \therefore\ r \end{array}$$

7. (4pts) Is the following argument valid? (Circle Yes / No)

$$\begin{array}{r} \neg s \rightarrow \neg t \\ s \\ \hline \therefore\ t \end{array}$$

8. (5pts) Write the negation of the following proposition so that (i) All quantifiers are to the left of negations (this means no $\neg\forall$ or $\neg\exists$), and (ii) No negations appear outside of a set of parentheses (this means no $\neg(\cdots)$):

$$\forall x\ \ (P(x) \rightarrow (\neg Q(x) \vee R(x)))$$

9. (8pts) Define $Q(x, y)$ to be the predicate "$x + 2y = xy$". Circle the truth value of the following statements. (Recall that $\mathbb{Z}$ is the set of integers.)

   (True / False) **(a)** $Q(1, -1)$

   (True / False) **(b)** $Q(0, 2)$

   (True / False) **(c)** $\forall y \in \mathbb{Z}\ \exists x \in \mathbb{Z}\ Q(x, y)$

   (True / False) **(d)** $\exists y \in \mathbb{Z}\ Q(3, y)$

10. (5pts) For this question, $F(A)$ is the predicate "A is a finite set," $S(A, B)$ is the predicate "A is a subset of B," and the domain of every quantifier is the universe of all sets. Translate the following statement into a concise, meaningful English sentence (Do not use "It is not the case that..."):

$$\neg\forall B \forall A(F(A) \wedge \neg F(B) \wedge S(A, B))$$

11. (4pts) The associative property of multiplication of the set of integers $\mathbb{Z}$ says that you can multiply three integers in arbitrary order and get the same result. Express this property as a quantified statement.

12. (8pts) Prove the following statement. When n is an integer, the following are equivalent:
(1) n^2 is even;
(2) $(n+1)^2$ is odd;
(3) n is even.

13. (8pts) Among a certain group of 35 people, exactly 1 person was born in the month of January. Prove that there is a month in which at least 4 people were born.

14. (8pts) Prove that $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$. There are several possible ways to do this. A Venn diagram can be helpful but is not a proof.

15. (4pts) Write down the power set of $\{b, \emptyset\}$.

16. (8pts) Use Venn diagrams to justify which relationship ($\subseteq$, $=$, or $\supseteq$) is valid for the following pair of sets. Write the correct operator in the blank.

    $B - C$ _____ $(B - A) - (C - A)$

[WORKSPACE]

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