PRINT Last name:________________ First name:________________

Signature:________________ Student ID:________________

Math 230 Exam 1, Spring 2009

1. (8pts) Determine whether or not $p \to (q \to r)$ is logically equivalent to $(p \to q) \to r$. Show your work or carefully describe your argument.

2. (8pts) For which rows of the truth table is the compound proposition $(p \oplus q) \to (q \leftrightarrow r)$ false?

p	q	r	
T	T	T	
T	T	F	
T	F	T	
T	F	F	
F	T	T	
F	T	F	
F	F	T	
F	F	F	

3. (6pts) The original statement is "If $1 + 1 = 3$ then $2 + 2 = 4$." Circle the correct truth value of each of the following statements:

   Contrapositive of the original statement (True / False)

   Converse of the original statement (True / False)

   Inverse of the original statement (True / False)

4. (4pts) Write the negation of the following statement (Do not write "It is not the case that ...").
   "I will go to the movies or read a book but not both."

5. (4pts) Is the following argument valid? (Circle Yes / No)

$$\begin{array}{r} p \to q \\ \neg p \\ \hline \therefore\ \neg q \end{array}$$

6. (4pts) Is the following argument valid? (Circle Yes / No)

$$\begin{array}{r} p \to \neg q \\ q \\ \hline \therefore\ \neg p \end{array}$$

7. (8pts) Among these 5 propositions are exactly 1 tautology and exactly 1 contradiction. Write $\mathbb{T}$ next to the tautology. Write $\mathbb{F}$ next to the contradiction. Do nothing for the rest of the propositions.

   _____**(a)** $p \vee q \vee r$

   _____**(b)** $p \vee (p \wedge \neg q) \vee \neg p$

   _____**(c)** $p \vee (q \wedge r)$

   _____**(d)** $q \to \neg q$

   _____**(e)** $(p \leftrightarrow q) \wedge (\neg p \leftrightarrow q)$

8. (8pts) Define $P(x, y)$ to be the predicate "$x + 2y = xy$". Circle the truth value of the following statements. (Recall that $\mathbb{Z}$ is the set of integers.)

   (True / False) **(a)** $P(0, 2)$

   (True / False) **(b)** $P(1, -1)$

   (True / False) **(c)** $\exists y \in \mathbb{Z}\ P(3, y)$

   (True / False) **(d)** $\forall y \in \mathbb{Z}\ \exists x \in \mathbb{Z}\ P(x, y)$

9. (5pts) Write the negation of the following proposition so that (i) All quantifiers are to the left of negations (this means no $\neg\forall$ or $\neg\exists$), and (ii) No negations appear outside of a set of parentheses (this means no $\neg(\cdots)$):

$$\exists x\ \ (P(x) \to (Q(x) \wedge \neg R(x)))$$

10. (4pts) The associative property of multiplication of the set of real numbers $\mathbb{R}$ says that you can multiply three real numbers in arbitrary order and get the same result. Express this property as a quantified statement.

11. (5pts) For this question, $F(A)$ is the predicate "A is a finite set," $S(A, B)$ is the predicate "A is a subset of B," and the domain of every quantifier is the universe of all sets. Translate the following statement into a concise, meaningful English sentence (Do not use "It is not the case that..."):

$$\neg\exists A\exists B(\neg F(A) \wedge F(B) \wedge S(A, B))$$

12. (8pts) Among a certain group of 27 people, exactly 2 people were born on Sunday. Prove that at least 5 people were born on the same day of the week.

13. (8pts) Prove the following statement. When n is an integer, the following are equivalent:
(1) n^2 is odd;
(2) $(n + 1)^2$ is even;
(3) n is odd.

14. (4pts) Write down the power set of $\{\emptyset, a\}$.

15. (8pts) Use Venn diagrams to justify which relationship ($\subseteq$, $=$, or $\supseteq$) is valid for the following pair of sets. Write the correct operator in the blank.

    $(A - C) - (B - C)$ _____ $A - B$

16. (8pts) Prove that $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$. There are several possible ways to do this. A Venn diagram can be helpful but is not a proof.

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