PRINT Last name:	First name:
Signature:	Student ID:

Math 230 Exam 1, Spring 2009

1. (8pts) Determine whether or not $p \to (q \to r)$ is logically equivalent to $(p \to q) \to r$. Show your work or carefully describe your argument.

2. (8pts) For which rows of the truth table is the compound proposition $(p \oplus q) \to (q \leftrightarrow r)$ false?

p	q	r	
T	Т	T	
Т	Т	F	
Т	F	Т	
Т	F	F	
F	Т	Т	
F	Т	F	
F	F	Т	
F	F	F	

3. (6pts) The original statement is "If 1 + 1 = 3 then 2 + 2 = 4." Circle the correct truth value of each of the following statements:

Contrapositive of the original statement (True / False)

Converse of the original statement (True / False)

Inverse of the original statement (True / False)

- 4. (4pts) Write the negation of the following statement (Do not write "It is not the case that ..."). "I will go to the movies or read a book but not both."
- 5. (4pts) Is the following argument valid? (Circle Yes / No)

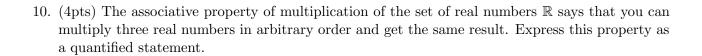
$$\begin{array}{c}
p \to q \\
\neg p \\
\hline
\vdots \quad \neg q
\end{array}$$

6. (4pts) Is the following argument valid? (Circle Yes / No)

$$\begin{array}{c} p \to \neg q \\ \hline & q \\ \hline \vdots & \neg p \end{array}$$

- 7. (8pts) Among these 5 propositions are exactly 1 tautology and exactly 1 contradiction. Write \mathbb{T} next to the tautology. Write \mathbb{F} next to the contradiction. Do nothing for the rest of the propositions.
 - $\underline{\hspace{1cm}}$ (a) $p \lor q \lor r$
 - $\underline{\hspace{1cm}}$ (b) $p \lor (p \land \neg q) \lor \neg p$
 - $\underline{\hspace{1cm}}$ (c) $p \lor (q \land r)$
 - $\underline{\hspace{1cm}}$ (d) $q \rightarrow \neg q$
 - $\underline{\hspace{1cm}}$ (e) $(p \leftrightarrow q) \land (\neg p \leftrightarrow q)$
- 8. (8pts) Define P(x,y) to be the predicate "x+2y=xy". Circle the truth value of the following statements. (Recall that $\mathbb Z$ is the set of integers.)
 - (True / False) (a) P(0,2)
 - (True / False) (b) P(1,-1)
 - (True / False) (c) $\exists y \in \mathbb{Z} \ P(3,y)$
 - (True / False) (d) $\forall y \in \mathbb{Z} \exists x \in \mathbb{Z} P(x,y)$
- 9. (5pts) Write the negation of the following proposition so that (i) All quantifiers are to the left of negations (this means no ¬∀ or ¬∃), and (ii) No negations appear outside of a set of parentheses (this means no ¬(···)):

$$\exists x \ (P(x) \to (Q(x) \land \neg R(x)))$$



11. (5pts) For this question, F(A) is the predicate "A is a finite set," S(A, B) is the predicate "A is a subset of B," and the domain of every quantifier is the universe of all sets. Translate the following statement into a concise, meaningful English sentence (Do not use "It is not the case that..."):

$$\neg \exists A \exists B (\neg F(A) \land F(B) \land S(A, B))$$

12. (8pts) Among a certain group of 27 people, exactly 2 people were born on Sunday. Prove that at least 5 people were born on the same day of the week.

13. (8pts) Prove the following statement. When n is an integer, the following are equivalent:

- (1) n^2 is odd;
- (2) $(n+1)^2$ is even;
- (3) n is odd.

- 14. (4pts) Write down the power set of $\{\emptyset, a\}$.
- 15. (8pts) Use Venn diagrams to justify which relationship (\subseteq , =, or \supseteq) is valid for the following pair of sets. Write the correct operator in the blank.

$$(A-C)-(B-C) \quad \underline{\hspace{1cm}} \quad A-B$$

16. (8pts) Prove that $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$. There are several possible ways to do this. A Venn diagram can be helpful but is not a proof.

[WORKSPACE]

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