PRINT Last name:____________________ First name:____________________

Signature:____________________ Student ID:____________________

Math 230 Exam 1, Spring 2008

Show work for full credit. Partial credit for good proof structure even if proof is not correct.

1. A couple of weeks ago, I used the logical equivalence $p \to (q \vee r) \equiv (p \wedge \neg q) \to r$ to simplify a proof in a paper. Prove that this actually is a logical equivalence.

2. Recall that a logical proposition involving only propositional variables, negations, and conjunctions (e.g., $p \wedge q \wedge \neg r$) is true for only one row of its truth table. Using this as a building block, find a proposition having the following truth table.

p	q	r	??
T	T	T	F
T	T	F	F
T	F	T	T
T	F	F	T
F	T	T	F
F	T	F	F
F	F	T	F
F	F	F	T

3. Write the converse, inverse, and contrapositive of the following: "The team wins if the quarterback can run."

   Contrapositive:

   Converse:

   Inverse:

4. Circle the negation of the proposition $\forall x\Big(\big(P(x) \vee Q(x)\big) \to \neg R(x)\Big)$.

   (a) $\forall x\big(\neg R(x) \to (P(x) \vee Q(x))\big)$
   (b) $\exists x\big(\neg P(x) \vee \neg Q(x) \vee \neg R(x)\big)$
   (c) $\forall x\big((\neg P(x) \wedge \neg Q(x)) \vee \neg R(x)\big)$
   (d) $\exists x\big((P(x) \vee Q(x)) \wedge R(x)\big)$
   (e) $\forall x\big(R(x) \to (\neg P(x) \wedge \neg Q(x))\big)$

5. Write $\mathbb{T}$ next to the tautology. Write $\mathbb{F}$ next to the contradiction. Do nothing for the rest of the propositions.

   _____**(a)** $p \vee (q \vee \neg p)$

   _____**(b)** $p \wedge (r \vee p)$

   _____**(c)** $(p \leftrightarrow q) \wedge (p \oplus q)$

   _____**(d)** $q \rightarrow \neg q$

   _____**(e)** $(p \oplus q) \wedge \neg p$

6. Suppose the variable x represents students, y represents courses, and $T(x, y)$ means "x is taking y". Next to the quantified statement, write the number of the equivalent English sentence.

___	$\forall y \exists x\ T(x, y)$	**(1)**	No student is taking any course.
___	$\exists x \forall y\ T(x, y)$	**(2)**	Every course is being taken by at least one student.
___	$\neg\big(\exists x \exists y\ T(x, y)\big)$	**(3)**	Some student is taking every course.
___	$\neg\big(\forall x \neg(\forall y\ \neg T(x, y))\big)$	**(4)**	Some students are taking no courses.

7. Explain the case for truth values of p and q that shows that the following argument is not valid.

$$\begin{array}{l} p \rightarrow q \\ \underline{q \qquad\quad} \\ \therefore p \end{array}$$

8. Prove the following statement. If n^3 is an even integer then n is even. (Hint: is direct, contrapositive or contradiction easier?)

9. Prove that the equation $4x^2 + y^2 = 14$ has no integer solutions.

10. Recall that by definition $A \oplus B = (A - B) \cup (B - A)$. Prove by set membership table that $A \oplus (B \oplus C) = (A \oplus B) \oplus C$.

A	B	C	

11. Draw two Venn diagrams (one for the left side, one for the right side) to justify which relationship, $\subseteq$, $=$, or $\supseteq$, is valid for the following pair of sets. Write the correct operator in the blank.

$A \cup (B \cap C)$ _____ $(A \cup B) \cap C$

12. In specifying a function, give the domain and codomain. Also give either the rule or clearly describe how the function is defined.

    (a) Give an example of a function which is strictly decreasing.

    (b) Give an example of a function which is (monotonic) decreasing but not strictly decreasing.

13. In the space provided, write whether or not the rule with the given domain and codomain describes a function, and if not give a brief reason why. Recall the natural numbers are $\mathbb{N} = \{0, 1, 2, \ldots\}$.

    (a) $f : \mathbb{N} \to \mathbb{N}$, where $f(n) = \sqrt{n}$.

    _______(Yes/No) Reason:

    (b) $h : \mathbb{R} \to \mathbb{R}$, where $h(x) = \sqrt{x}$.

    _______(Yes/No) Reason:

    (c) $g : \mathbb{N} \to \mathbb{N}$, where $g(n)$ equals any integer greater than n .

    _______(Yes/No) Reason:

14. Prove or disprove that the function $g : \mathbb{N} \to \mathbb{N}$ defined by $g(n) = 5n + 2$ is one-to-one.