PRINT Last name:________________ First name:________________

Signature:________________ Student ID:________________

Math 152 Final Exam, Fall 2005

Instructions. You must show the mathematical steps of the solution in order to receive full credit; writing only the answer will receive no credit. Elaborate written explanations for each step are not required, but it is a good idea to write brief justifications for the major steps. An incorrect solution with a correct written explanation might receive partial credit.

Conditions. No calculators, computers, notes, books, or scratch paper. By writing your name on the exam you certify that all work is your own, under penalty of all remedies outlined in the IIT student rules. **Please do not talk until you are away from the room.**

NOTE: The topics may not be in order either of increasing difficulty or of the order they were covered in the course.

Time limit: 2 hours (strict).

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POSSIBLY USEFUL FORMULAS

$\sec^2 x = \tan^2 x + 1$ $\qquad M_n = \Delta x \left[f\left(\frac{x_0+x_1}{2}\right) + f\left(\frac{x_1+x_2}{2}\right) + \cdots + f\left(\frac{x_{n-1}+x_n}{2}\right)\right]$

$\cos^2 x = \frac{1+\cos 2x}{2}$ $\qquad T_n = \frac{\Delta x}{2}\left[f(x_0) + 2f(x_1) + \cdots + 2f(x_{n-1}) + f(x_n)\right]$

$\int \frac{dx}{1+x^2} = \tan^{-1} x + C$ $\qquad \int \tan x \, dx = -\ln|\cos x| + C$

$PV = nRT$ $\qquad |E_M| < \frac{K(b-a)^3}{24n^2} \quad (K \geq f''(x))$

$F = \rho g A d$ $\qquad |E_T| < \frac{K(b-a)^3}{12n^2} \quad (K \geq f''(x))$

$|R_n(x)| \leq \frac{M}{(n+1)!}|x-a|^{n+1}$ $\qquad \mathbf{n} \cdot (\mathbf{r} - \mathbf{r}_0) = 0$

$S_n = \frac{\Delta x}{3}\left[f(x_0) + 4f(x_1) + 2f(x_2) \cdots + 2f(x_{n-2}) + 4f(x_{n-1}) + f(x_n)\right]$

$|E_S| < \frac{K(b-a)^5}{180n^4} \quad (K \geq f^{(4)}(x))$

$\int_{n+1}^{\infty} f(x)\,dx \leq R_n \leq \int_n^{\infty} f(x)\,dx$ $\qquad \int \frac{1}{\sqrt{1-x^2}}\,dx = \sin^{-1} x + C$

$\frac{1}{1+x^2} = \sum_{n=0}^{\infty} (-1)^n x^{2n}$ $\qquad \frac{d}{dx}(\sec^{-1} x) = \frac{1}{x\sqrt{x^2-1}}$

$|s - s_n| \leq |a_{n+1}|$ $\qquad \sum_{n=0}^{\infty} x^n = \frac{1}{1-x}$

$\sin 2x = 2\sin x \cos x$ $\qquad \mathrm{Vol} = \int_a^b \pi\left[(f(x))^2 - (g(x))^2\right]\,dx$

SHOW WORK FOR FULL CREDIT NO CALCULATORS

1. Compute the Taylor series centered at $a = 3$ for the function $f(x) = e^x$.

2. A geometric series has first term $a = 2$. Find the ratio r of the series so that its sum is 6.

3. Find the **radius** of convergence of the power series $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n (x+4)^n}{n^2 \cdot 3^n}$.

4. Let $w = 1 - 2i$ and $z = 3 + 4i$.
   (i) Find the conjugate $\overline{z}$ of z.
   (ii) Find the modulus $|z|$ of z.
   (iii) Find $\dfrac{w}{z}$ in the form $a + bi$.

5. Find the equation of the line tangent to the following curve at the point $(x, y) = (1, 1)$:

$$\begin{aligned} x &= e^t \\ y &= (t-1)^2 \\ & -\infty < t < \infty \end{aligned}$$

6. Compute the limit of the sequence $\left\{ \dfrac{n^2}{3^n} \right\}_{n=1}^{\infty}$ or determine how it diverges.

7. Determine if the following integral converges or diverges, and give its value if it converges.

$$\int_0^\infty xe^{-x^2}\, dx\, .$$

8. A radioactive substance decays so that after 3 hours, exactly 90% of the original amount remains. Find the time at which 40% of the substance remains.

9. Find a particular solution y to the differential equation $\dfrac{dy}{dx} = -5(y-40)$ which satisfies $y(0) = 50$.

10. Differentiate the function $y = x^{\cos x}$.

11. Fully simplify the expression $\sin^{-1}\left(\cos\left(\frac{5\pi}{6}\right)\right)$.

12. Find the integral $\int x^2 \cos x \, dx$.

13. Use a trigonometric substitution of the form $x = f(\theta)$ to **fully convert** the following integral into the variable θ, but **do not integrate**:

$$\int_{-2}^{2} \frac{x^2}{\sqrt{16 - x^2}}\, dx\,.$$

14. Use 2 terms of an appropriate power series to estimate $\displaystyle\int_0^{0.1} \tan^{-1}(x^2)\, dx$. Give, with justification, a bound on the error in this estimate.