Fall 2004
Math 152
EXAM 3
Test Form A

PRINT: **Last Name:** ______________________________ **First Name:** ______________________

Signature: ____ **ID:** _________________________

Instructor's Name: ____ **Section #** _____________

Instructor use only.

Part I(40)		
Part II(60)	11	
	12	
	13	
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	15	
	16	
	17	
	18	
Exam Total(100)		

INSTRUCTIONS

1. Write your **name**, **exam form letter**, and **section number** on your scantron.
2. In **Part I**, clearly mark exactly one answer on your scantron for each problem. The scantron will not be returned.
3. In **Part II**, show work in order to expect full credit. Partial credit is possible for positive progress towards the correct solution.
4. By signing your name to the exam you certify that all work is your own and that no notes, calculators, or other sources were used.
5. **Be sure you have all 9 pages with problems 1-18.**

PART I: MULTIPLE-CHOICE PROBLEMS

Part 1: Multiple Choice Problems. Each problem is worth 4 points each: NO partial credit will be given. Calculators may not be used.

1. The series $\displaystyle\sum_{n=1}^{\infty} n^q$ converges precisely when

   (a) $q > 0$ (b) $q > -1$ (c) $|q| < 1$ (d) $q < 0$ (e) $q < -1$

2. Consider the two series (1) $\displaystyle\sum_{n=1}^{\infty} ne^{-n^2}$ and (2) $\displaystyle\sum_{n=1}^{\infty} n^{-1}e^{n^2}$.

   (a) Convergence of at least one cannot be decided
   (b) Both converge
   (c) Both diverge
   (d) (1) diverges and (2) converges
   (e) (1) converges and (2) diverges

3. The series $\displaystyle\sum_{n=2}^{\infty} (-1)^n \frac{\sqrt{n}}{\ln n}$ can be shown to be

   (a) convergent by Limit Comparison Test
   (b) convergent but not absolutely convergent
   (c) divergent by the Ratio Test
   (d) absolutely convergent
   (e) divergent because $\{a_n\}$ does not converge to 0

4. Which of the following sequences is neither an increasing sequence nor a decreasing sequence?

(a) $\left\{\frac{1}{n}\right\}_{n=1}^{\infty}$ (b) $\left\{\tan\left(\frac{1}{n^2}\right)\right\}_{n=1}^{\infty}$ (c) $\left\{\sin\left(\frac{\pi n}{6}\right)\right\}_{n=1}^{\infty}$ (d) $\left\{\frac{\ln n}{n}\right\}_{n=3}^{\infty}$ (e) $\{n!\}_{n=1}^{\infty}$

5. The series $\displaystyle\sum_{n=1}^{\infty} \frac{n^2}{n^3+\sqrt{n^9+1}}$ can be shown to be

(a) convergent by the Limit Comparison Test
(b) convergent by the Ratio Test
(c) divergent by the Ratio Test
(d) divergent by the Limit Comparison Test
(e) convergent because $\{a_n\}$ converges to 0

6. The sequence defined by $a_1 = 2$ and $a_{n+1} = \dfrac{2-a_n}{1-a_n}$ for all $n \geq 1$

(a) converges to 0
(b) converges to 1
(c) converges to 2
(d) diverges to ∞
(e) diverges, but does not diverge to ∞

7. Find all of the values of x in the interval $[0, \pi]$ for which the series $\sum_{n=1}^{\infty} (\cos x)^n$ converges.

(a) $\left(\frac{\pi}{4}, \frac{3\pi}{4}\right]$ (b) $(0, \pi)$ (c) $\left(\frac{\pi}{6}, \frac{2\pi}{3}\right]$ (d) $(0, \pi]$ (e) $\left[\frac{\pi}{6}, \frac{2\pi}{3}\right)$

8. The sum of the series $\sum_{n=1}^{\infty} \frac{(-3)^n}{4^{n+1}}$ is equal to

(a) 3/4 (b) $-3/28$ (c) $-3/4$ (d) 3/28 (e) ∞

9. The series $\displaystyle\sum_{n=1}^{\infty} \frac{n!(2n)!}{(3n)!}$

(a) converges because it is a p-series
(b) converges by Ratio Test
(c) diverges by Ratio Test
(d) diverges by Limit Comparison Test
(e) diverges because $\{a_n\}$ does not converge to 0

10. Use power series to compute $\displaystyle\lim_{x\to 0} \frac{x^5 \cdot e^x}{\sin(x) - x + \frac{1}{6}x^3}$.

(a) ∞ (b) 0 (c) 1 (d) 120 (e) 200

PART II. WORKOUT PROBLEMS

Part 2: Work-Out Problems. Partial credit is possible. Lack of detail or clarity is subject to penalty. CLEARLY INDICATE any work continued elsewhere. NO CALCULATORS.

11. **(8pts)** Determine whether or not the series $\displaystyle\sum_{n=2}^{\infty} \frac{(-1)^n \sqrt{n}}{n^2 - n}$ is absolutely convergent.

12. **(8pts)** If the series $\displaystyle\sum_{n=1}^{\infty} a_n$ has nth partial sum $s_n = 1 + \dfrac{n+1}{2n-1}$,

    (a) Find the sum of the series.

    (b) find the nth term a_n of the series.

13. **(8pts)** Estimate the sum of the series $\sum_{n=1}^{\infty} \frac{(-1)^n}{(n-1)^2+2}$ to within $\frac{1}{18}$. Your estimate need not be fully simplified.

14. **(8pts)** Find the interval of convergence of the power series $\sum_{n=0}^{\infty} \frac{(-1)^n(x-3)^n}{2^n\sqrt{n+1}}$.

15. **(8pts)** Evaluate $\displaystyle\int_0^1 \frac{\tan^{-1}(x)}{x}\,dx$, expressing your answer as an infinite series.

$\left(\text{Hint: the Taylor series of } \tan^{-1}(x) \text{ is } \displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{2n+1}\ .\right)$

16. **(6pts)** **(a)** Write down the Maclaurin series for e^x.

(b) Write down the Maclaurin series for $\cos\left(x^4\right)$.

(c) Compute the first 5 <u>nonzero</u> terms of the Maclaurin series of $f(x) = e^x \cdot \cos\left(x^4\right)$.

17. **(6pts)** Find a power series for $\dfrac{x^2}{1+x^4}$.

18. **(8pts)** Compute the degree 3 Taylor polynomial $T_3(x)$ centered at $a = 0$ of the function

$$f(x) \;=\; \int_0^x e^{t^2}\,dt\,.$$